17. V. A. Ryabin, Thermodynamical Properties of Substances [in Russian], Leningrad (1977).
18. A. Lietoila, Laser Processing of High-Dose Ion Implanted Si: The Solid Phase Regime, Thesis, Stanford (1981), p. 120.
19. A. E. Sheindlin (ed.), Radiation Properties of Solids, Handbook [in Russian], Moscow (1974).
20. D. B. Zvorykin and Yu. I. Prokhorov, Application of Radiant Infrared Heating in Electronic Engineering [in Russian], Moscow (1980).
21. A. S. Okhotin and A. S. Pushkarskii, Thermophysical Properties of Semiconductors [in Russian], Moscow (1972).
22. C. W. White and P. S. Peercy (eds.), Laser and Electron Beam Processing of Materials, New York (1980), pp. 6-9.
23. A. Bhattacharya, B. G. Streetman, and K. Hess, J. Appl. Phys., 52, No. 5, 3611-3617 (1981).
24. N. M. Belyaev and A. A. Ryadno, Methods of the Theory of Heat Conduction [in Russian], Moscow (1982).

POWER-LAW APPROXIMATION OF TRANSFER FUNCTION IN NONSTEADY HEAT- AND MASS-TRANSFER PROBLEMS

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A power-law approximation of the transfer function for heat- and mass-transfer problems is proposed, enabling the temperature at a fixed point to be found to within 2%.

The method of imaginary-frequency characteristics (IFC) proposed in [1] and developed for heat- and mass-transfer problems in [2, 3] consists in replacing the accurate transform F(p) in the region of real p by some approximate function $\Phi(p)$, the inverse of which may readily be found. It was shown in [1] that, for weakly oscillatory systems, the error of the inverse is practically equal to the error of the approximation of the transform. In [3], the IFC method was used to construct analytical solutions of the thermal-conductivity problems for an infinite plate, cylinder, and sphere. The accuracy of the results is 1-3%. However, the use of piecewise-rational functions for $\Phi(p)$ in [3] leads to certain difficulties in finding the coefficients of the approximation [4]. Therefore, the question of using functions of a different form for this purpose arises. Investigations show that, for monotonic dependences F(p) such as the transfer functions in heat- and mass-transfer problems, a sufficiently effective approximation is a power-law function

$$\Phi(p) = \left(\frac{A}{A+p}\right)^{L} \exp(-Tp). \tag{1}$$

In comparison with the piecewise-rational function, this approximation permits more effective description of both slowly and rapidly decreasing dependences. In addition, the parameters A, L, T in [1] may be found by the least-squares method, which leads to fairly complex nonlinear expressions in the case of the piecewise-rational function [1].

The scheme of the least-squares method for the case of [1] is now outlined. Suppose that F(p) is normalized to F(0) = 1, and its value is known at N points. Then, the parameters of the approximation are found from the condition of a minimum of the sum of squares of the deviations of ln F(p) and ln $\Phi(p)$

$$Q = \sum_{i=1}^{N} \left[\ln F(p_i) - L \ln \frac{A}{p_i + A} + Tp_i\right]^2 \to \min. \tag{2}$$

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Finding the partial derivatives with respect to the parameters and equating them to zero, a system of three equations is obtained, yielding, after simple transformations, a nonlinear equation for determining A

$$S_1S_2S_6S_8 - S_1^2S_2S_7 - S_1S_5S_6^2 - S_1S_3S_4S_8 + S_1^2S_5S_8 + S_1S_3S_6S_7 = 0, \tag{3}$$

$$S_1 = \sum_{i=1}^{N} p_i^2;\ S_2 = \sum_{i=1}^{N} \ln F(p_i) \ln \frac{A}{p_i + A};\ S_3 = \sum_{i=1}^{N} p_i \ln F(p_i);$$

$$S_4 = \sum_{i=1}^{N} \frac{p_i}{p_i + A};\ S_5 = \sum_{i=1}^{N} \frac{p_i}{p_i + A} \ln F(p_i);\ S_6 = \sum_{i=1}^{N} p_i \ln \frac{A}{p_i + A};$$

$$S_7 = \sum_{i=1}^{N} \frac{p_i}{p_i + A} \ln \frac{A}{p_i + A};\ S_8 = \sum_{i=1}^{N} \left(\ln \frac{A}{p_i + A} \right)^2.$$

The terms in Eq. (3) depend on A. The parameter A is determined as the square root of Eq. (3) by the well-known method of dividing the segment in two. After finding A, L and T are calculated from formulas in which S_i are known for the given A

$$L = (S_3S_4 - S_1S_5)/(S_6S_4 - S_1S_7), \tag{4}$$

$$T = (LS_6 - S_3)/S_1. \tag{5}$$

Taking the inverse of Eq. (1), the desired function is obtained

$$\varphi(\tau) = \frac{1}{\Gamma(L)} A^L (\tau - T)^{L-1} \exp[-A(\tau - T)]. \tag{6}$$

As an example, consider the thermal conductivity of a symmetric one-dimensional plate with boundary conditions of the first kind, the solution of which in the Laplace-transform region takes the form [5]

$$\overline{\Theta}(x,\ p) = \overline{f}(p) \frac{\operatorname{ch}(\sqrt{p}x)}{\operatorname{ch}\sqrt{p}}, \tag{7}$$

where

$$\overline{f}(p) = \int_0^\infty f(\tau) \exp(-p\tau)\, d\tau,\ \overline{\Theta}(x,\ p) = \int_0^\infty \Theta(x,\ \tau) \exp(-p\tau)\, d\tau.$$

For inversion, $\operatorname{ch}(\sqrt{px})/\operatorname{ch}\sqrt{p}$ is approximated by a function of the form in Eq. (1) with a fixed value $x = x_0$. Then the approximate solution of the problem is obtained by calculating the convolution of $f(\tau)$ and $\phi(\tau)$

$$\Theta(x_0,\ \tau) = \begin{cases} \dfrac{1}{\Gamma(L)} \displaystyle\int_0^{A(\tau-T)} f\left(\tau - \frac{v}{A} - T\right) v^{L-1} \exp(-v)\, dv, & \tau > T, \\ 0, & \tau \leqslant T. \end{cases} \tag{8}$$

In the case of arbitrary $f(\tau)$, numerical integration may be used for calculation by Eq. (8); this is considerably simpler than solving the problem by finite-difference methods or calculating the Duhamel integral using the accurate solution.

If the temperature at the boundary changes discontinuously, $f(\tau) = 1$, it follows from Eq. (8) that

$$\Theta(x_0,\ \tau) = \begin{cases} \dfrac{1}{\Gamma(L)} \displaystyle\int_0^{A(\tau-T)} v^{L-1} \exp(-v)\, dv, & \tau > T, \\ 0, & \tau \leqslant T. \end{cases} \tag{9}$$

The solution of the analogous problem for a cylinder and a sphere takes the form in Eq. (8) but with different values of the parameters.

Equation (9) constitutes an incomplete gamma function, the properties of which are well known; its values are tabulated in [6]. More accurate values of the incomplete gamma function may be obtained on a computer using well-known expansions [6].

To test the accuracy of the solution, the temperature is calculated in the middle of a plate with $f(\tau) = 1$, comparing the results with accurate data [5]. Analysis shows that the parameters and accuracy of the approximation depends on the choice of the range of variation of parameter p in IFC: increase of this range permits increase in the accuracy of the re-

TABLE 1. Approximation of the Function $1/\mathrm{ch}\sqrt{p}$

p	$1/\mathrm{ch}\sqrt{p}$	$\Phi(p)$	$\varepsilon\cdot10^4$, %
0	1	0,999998	1,54972
0,01	0,9995	0,999499	1,07342
0,01	0,995021	0,995021	—5,99029
0,05	0,97551	0,975516	—5,25468
0,1	0,952002	0,9520011	—9,82974
0,25	0,886819	0,886831	—1,32407
0,5	0,793278	0,793279	—1,57788
1	0,648054	0,648039	2,41894
2	0,459098	0,459104	—1,18145
3	0,343103	0,343101	5,12481

TABLE 2. Approximate and Accurate Solutions of the Problem of Plate Cooling

Fo	Θ	Θ_1	ε, %
0,06	0,9922	1	—0,786136
0,08	0,9752	0,994179	—1,94618
0,1	0,9493	0,955478	—0,650848
0,12	0,9175	0,915543	0,213264
0,14	0,8824	0,876075	0,716776
0,16	0,8458	0,83757	0,972991
0,18	0,8088	0,800248	1,05733
0,2	0,7723	0,764213	1,04709
0,22	0,7367	0,729512	0,975633
0,24	0,7022	0,696159	0,860253
0,26	0,669	0,664147	0,725466
0,28	0,6372	0,633455	0,587749
0,3	0,6088	0,604055	0,779335
0,32	0,5778	0,575914	0,32635
0,34	0,55	0,548994	0,182835
0,36	0,5236	0,523256	0,0657632
0,38	0,4985	0,498657	—0,031554
0,4	0,4745	0,475158	—0,138648
0,44	0,4299	0,43129	—0,323257
0,48	0,3895	0,391325	—0,468444
0,52	0,3529	0,354948	—0,580347
0,56	0,3198	0,321863	—0,644944
0,6	0,2897	0,291788	—0,720729
0,64	0,2625	0,264466	—0,749134
0,68	0,2378	0,239656	—0,780456
0,72	0,2155	0,217134	—0,758348
0,76	0,1952	0,196697	—0,766998
0,8	0,1769	0,178159	—0,711465
0,84	0,1602	0,161345	—0,714586
0,88	0,1452	0,146101	—0,620564
0,92	0,1315	0,132282	—0,594607
0,96	0,1192	0,119757	—0,46758
1	0,108	0,108408	—0,377593
1,04	0,0978	0,0981254	—0,332724
1,08	0,0886	0,0888112	—0,238335
1,12	0,0803	0,0803743	—0,0925338
1,16	0,0728	0,0727331	0,0918837
1,2	0,0659	0,0658143	0,130018
1,6	0,0246	0,0241467	1,84253
2	0,0092	0,008823	4,09158

sults for small Fourier number Fo but, on the other hand, the error becomes larger for Fo ~ 1. Decrease in the range of p yields a uniformly suitable approximation for all practically significant values of Fo. Results of approximating the transfer function $1/\mathrm{ch}\sqrt{p}$ when $0 \leq p \leq 3$ are given in Table 1. Table 2 gives the relative excess temperature Θ at the center of the plate according to the data of [5] and the values of Θ_1 calculated from Eq. (9). The parameter values in this case are: A = 2.55685; L = 1.08242; T = 0.0765.

It is evident from Table 2 that the error exceeds 1% only at certain points.

Calculations show that increasing the number of points in calculating the parameters A, L, T has no pronounced influence on the results.

Comparing the error of the approximation and the accuracy of the results in Tables 1 and 2, it may be concluded that these errors are only approximately equal up to values of ~1%, contrary to [1, 3], and further improvement in accuracy of the approximation does not yield adequate improvement in the final results.

Note also that the dependence of A, L, T on the choice of the range of p may be used to increase the accuracy of the results in the range of Fo which is of interest.

NOTATION

p, parameter of Laplace transformation; $\Phi(p)$, approximating function; A, L, T, parameters of approximation; τ, time; $\phi(t)$, inverse of $\Phi(p)$; $\Theta(x, \tau)$ dimensionless temperature; $\bar{\Theta}(x, p)$, transform of dimensionless temperature; $f(\tau)$, temperature at plate surface; $\bar{f}(p)$, transform of $f(\tau)$; Fo, Fourier number; ε, error of approximation; $\Gamma(L)$, gamma function.

LITERATURE CITED

1. A. I. Orurk, New Methods of Synthesizing Linear and Some Nonlinear Dynamic Systems [in Russian], Moscow (1965).
2. A. S. Trofimov, Inzh.-fiz. Zh., 49, 671-675 (1985).
3. A. S. Trofimov, A. V. Kozlov, E. Yu. Kovalenko, and E. M. Satsko, Inzh-fiz. Zh., 49, No. 3, 513 (1985); Paper 2299-85 Deposited at VINITI [in Russian], Moscow (1985).
4. A. S. Trofimov and S. A. Pas', Inzh.-fiz. Zh., 52, No. 6, 1021 (1989); Paper 8186-V86 Deposited at VINITI [in Russian], Moscow (1986).
5. A. V. Lykov, Theory of Thermal Conductivity [in Russian], Moscow (1967).
6. M. Abramovits and I. Stigan (eds.), Handbook on Special Functions [in Russian], Moscow (1979).

NUMERICAL INVESTIGATION OF THE PROCESS OF VENTILATIVE DRYING OF A PIPELINE

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A procedure for calculating the process of drying a pipeline with a desiccant is described that enables one to predict the time required for the complete removal of moisture. The influence of various factors on the rate of the process is analyzed without allowance for the temperature drop due to the heat consumed in evaporation.

Before putting into operation the pipelines designed for transporting high-quality petroleum products, as well as for supplying carbon dioxide to strata, it is necessary to dry the inner surface of the pipeline. (Otherwise, hydrates of hydrocarbons may form, resulting in obstruction of the pipes, measurement instruments, valves, etc., and hence in failures.) In the transportation of jet fuel, the presence of moisture is entirely unacceptable. The standard of the Petroleum Institute of the USA, for example, establishes a molar concentration of moisture in jet engine fuel of about 15 parts per million. A special feature of pipelines for pumping CO_2 is the need to carefully rid them of traces of water after a hydraulic test, since highly corrosive carbonic acid is formed in the interaction of CO_2 with water.

Most of the water that remains in a pipeline after a hydraulic test can be removed by successively passing a series of cylindrical or spherical inserts through the pipeline. After this operation, a small amount of water usually remains on the inner surface of the pipeline (the thickness of the water film may range from 50 to 100 μm), which can be removed by: a) ventilative drying; b) vacuum drying; c) absorptive drying.

In the present paper we investigate the process of ventilative drying (a fairly common means of removing moisture by means of a desiccant passed through the pipeline for a certain time). Air can be used as the desiccant in this case (it is exhausted directly into the atmosphere at the exit from the pipe, and at the entrance it is dehydrated and compressed) or another dry gas. A pipeline is considered to be ready for operation if the water vapor concentration in the desiccant at the exit from the pipe reaches a given minimum value;

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